Automatic Control Theory

CSE 322

Lec. 10

Types of Controllers

Quiz

- **By using Routh's criterion, investigate the stability of the control system:**

$$4\frac{d^5y(t)}{dt^5}+16\frac{d^4y(t)}{dt^4}+37\frac{d^3y(t)}{dt^3}+74\frac{d^2y(t)}{dt^2}+84\frac{dy(t)}{dt}+40y(t)=80r(t)$$

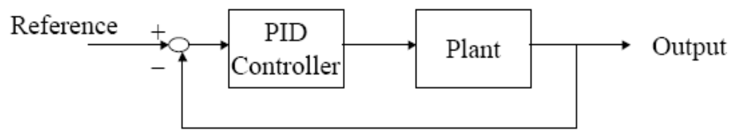
- Calculate the values of the system poles.

Lecture Outline

- Introduction to PID
- Modes of Control
  - On-Off Control
  - Proportional Control
  - Proportional + Integral Control
  - Proportional + Derivative Control
  - Proportional + Integral + Derivative Control

Introduction

- PID Stands for
  - P → Proportional
  - I → Integral
  - D → Derivative



Introduction

- The usefulness of PID controls lies in **their general applicability** to most control systems.
- In particular, when the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful.
- In the field of process control systems, it is well known that the basic and modified PID control schemes have proved their usefulness in **providing satisfactory control**, although in many given situations **they may not provide optimal control.**

Introduction

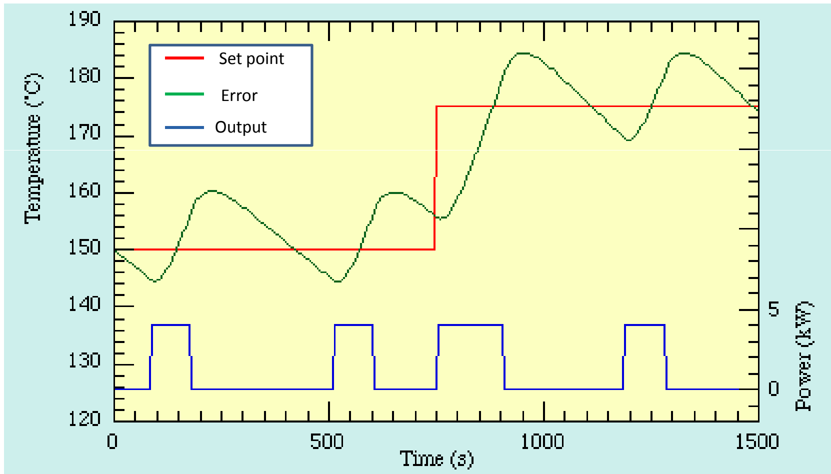
- It is interesting to note that **more than half of the industrial controllers** in use today are **PID controllers.**

Four Modes of Controllers

- Each mode of control has specific advantages and limitations.
  - On-Off (Bang Bang) Control
  - Proportional (**P**)
  - Proportional plus Integral (**PI**)
  - Proportional plus Derivative (**PD**)
  - Proportional plus Integral plus Derivative (**PID**)

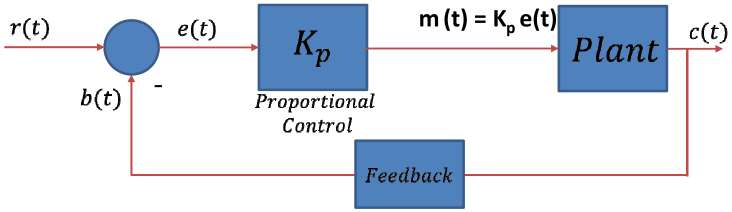
On-Off Control

- This is the simplest form of control.



Proportional Control (P)

- In ***proportional*** *mode*, there is a continuous linear relation between value of the controlled variable and position of the final control element.



- Output of proportional controller is

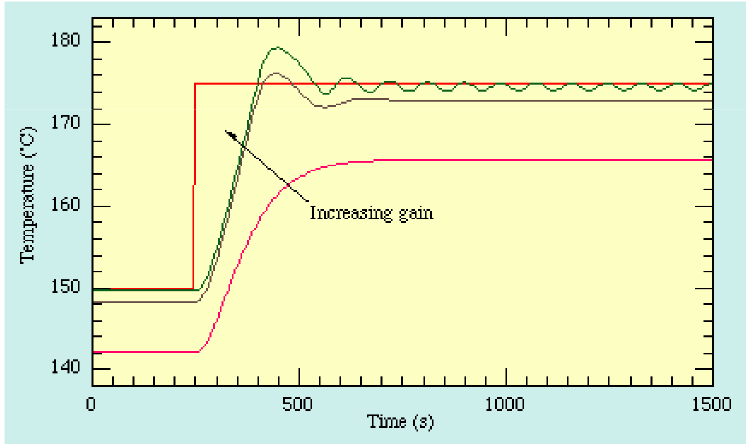
$$m(t) = K_p e(t)$$

- The transfer function can be written as

$$\frac{M(s)}{E(s)} = K_P$$

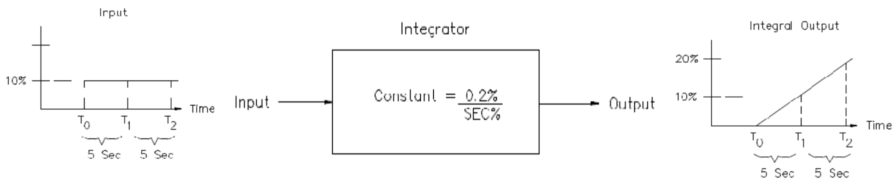
Proportional Controllers (P)

- As the gain is increased the system responds faster to changes in set-point but becomes progressively underdamped and eventually unstable.



Proportional Plus Integral Controllers (PI)

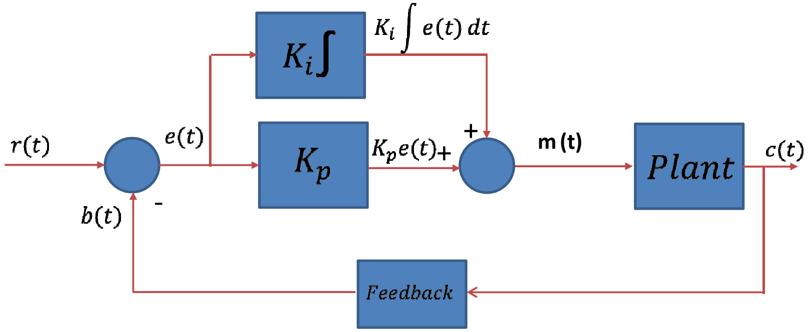
- Integral control describes a controller in which the output rate of change is dependent on the magnitude of the input.
- Specifically, a smaller amplitude input causes a slower rate of change of the output.



Proportional Plus Integral Controllers (PI)

- The major advantage of integral controllers is that they have the unique ability to **return the controlled variable back to the exact set point following a disturbance.**
- Disadvantages of the integral control mode are that it responds **relatively slowly to an error signal** and that it can initially allow a large deviation at the instant the error is produced.
- **This can lead to system instability** and cyclic operation. For this reason, **the integral control mode is <u>not normally used alone</u>**, but is combined with another control mode.

Proportional Plus Integral Control (**PI**)



$$\mathbf{m\,(t)} = K_p e(t) + K_i \int e(t)dt$$

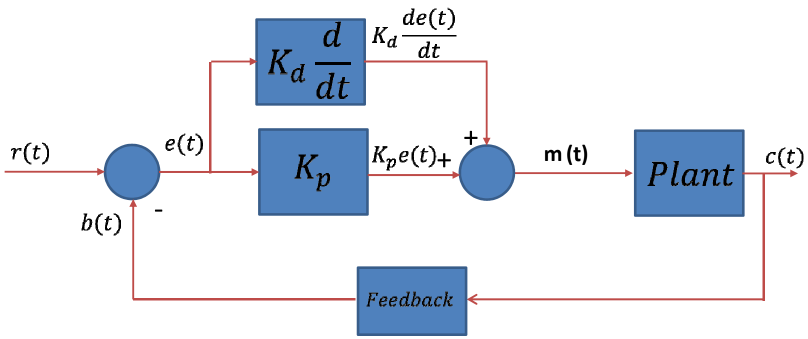
Proportional Plus Integral Control (**PI**)

$$\mathbf{m\,(t)} = K_p e(t) + K_i \int e(t)dt$$

- The transfer function can be written as

$$\frac{M(S)}{E(s)} = K_p + K_i \frac{1}{s}$$

Proportional Plus derivative Control (**PD**)



$$\mathbf{m\,(t)} = K_p e(t) + K_d \frac{de(t)}{dt}$$

Proportional Plus derivative Control (**PD**)

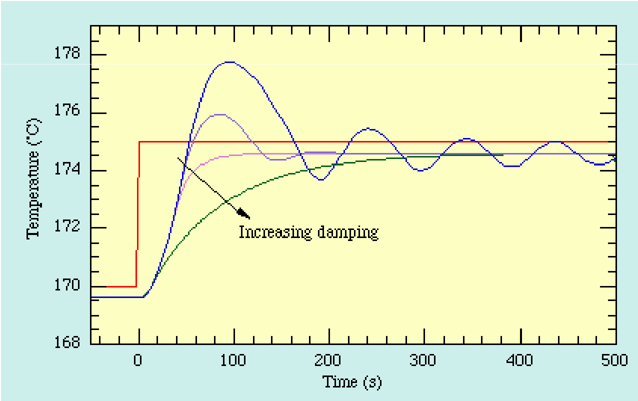
$$\mathbf{m\,(t)} = K_p e(t) + K_d \frac{de(t)}{dt}$$

- The transfer function can be written as

$$\frac{M(S)}{E(s)} = K_p + K_d s$$

Proportional Plus derivative Control (PD)

- The **stability and overshoot problems that arise when a proportional controller is used at high gain** can be mitigated by adding a term proportional to the time-derivative of the error signal. The value of the damping can be adjusted to achieve a critically damped response.

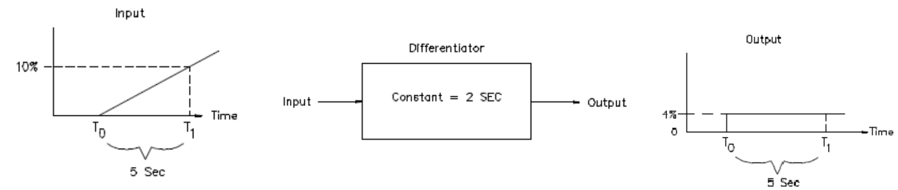


Proportional Plus derivative Control (PD)

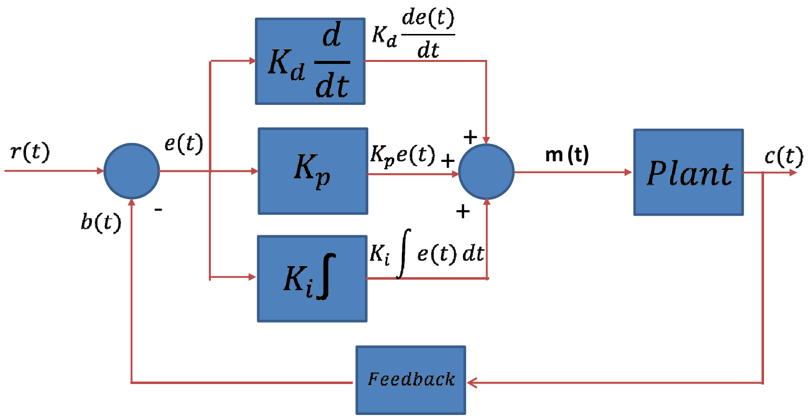
- **The higher the error signal rate** of change, **the sooner the final control element is positioned to the desired value**.

- **The added derivative action reduces initial overshoot** of the measured variable, and therefore aids in stabilizing the process sooner.

- This control mode is called proportional plus derivative (PD) control because the **derivative section responds to the rate of change of the error signal**



Proportional Plus Integral Plus Derivative Control (PID)



$$m(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \frac{de(t)}{dt}$$

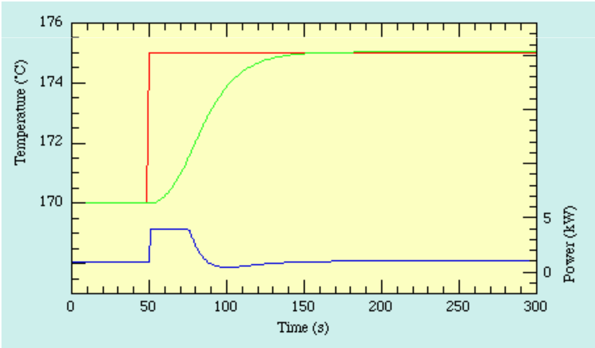
Proportional Plus Integral Plus Derivative Control (PID)

$$m(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \frac{de(t)}{dt}$$

$$\frac{M(S)}{E(s)} = K_p + K_i \frac{1}{s} + K_d\, s$$

Proportional Plus Integral Plus Derivative Control (PID)

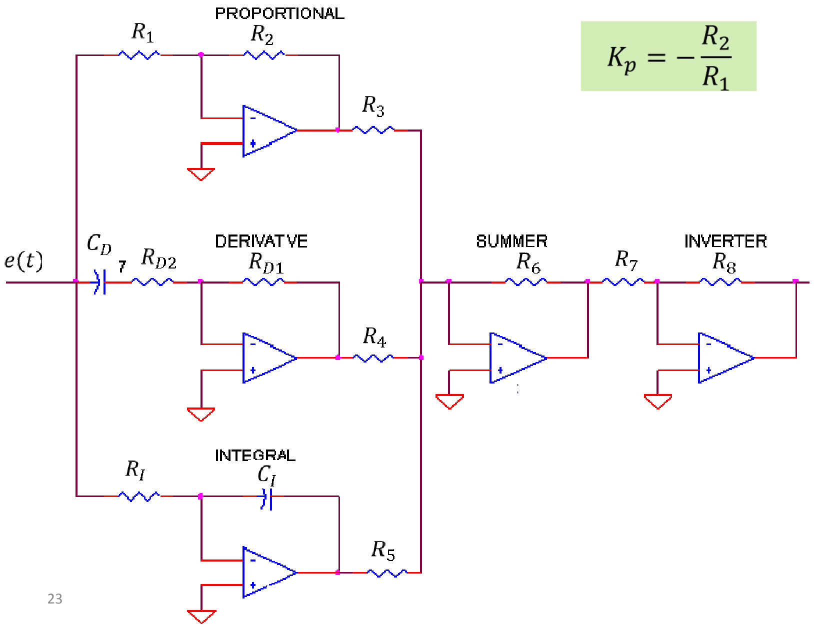
- Although **PD control** deals nearly with the overshoot and ringing problems associated with proportional control it **does not cure the problem with the steady-state error**. Fortunately it is possible to eliminate this while using relatively low gain **by adding an integral** term to the control function which becomes



The Characteristics of P, I, and D controllers

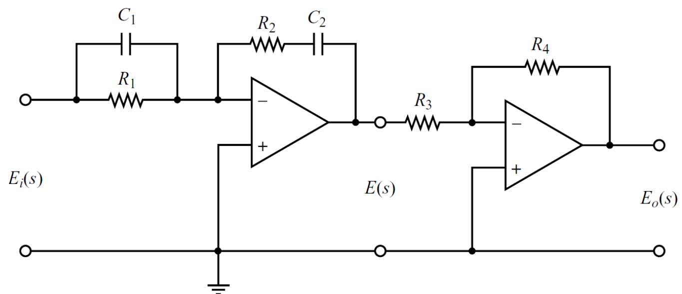
CONTROLLER	RISE TIME	OVERSHOOT	SETTLING TIME	S.S. ERROR
P ↑	Decrease	Increase	Small Change	**Decrease**
I	Decrease	Increase	Increase	Eliminate
D	Small Change	Decrease	Decrease	Small Change

Electronic PID Controller



$$K_p = -\frac{R_2}{R_1}$$

Electronic PID Controller



$$\frac{E_o(s)}{E_i(s)} = \frac{R_4}{R_3}\frac{(R_1C_1s+1)(R_2C_2s+1)}{R_2C_2s}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{R_4R_2}{R_3R_1}\left(\frac{R_1C_1+R_2C_2}{R_2C_2} + \frac{1}{R_2C_2s} + R_1C_1s\right)$$

Tips for Designing a PID Controller

1. Obtain an open-loop response and determine what needs to be improved
2. Add a **proportional** control to improve the **rise time**
3. Add a **derivative** control to improve the **overshoot**
4. Add an **integral** control to eliminate the **steady-state error**
5. Adjust each of K_p, K_i, and K_d until you obtain a desired overall response.

- Lastly, please keep in mind that you do not need to implement all three controllers (proportional, derivative, and integral) into a single system, if not necessary. For example, if a **PI** controller gives a good enough response (like the above example), then you don't need to implement derivative controller to the system. Keep the controller as simple as possible.

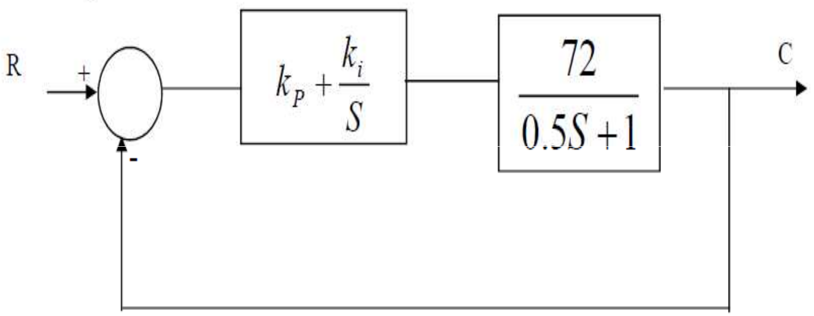
Notes

- For any controller design you must find

1. The Controller Type.
2. Its Gain values.

Example - 1

1. Design a PI controller to the shown system which has a maximum peak overshoot of 0.01 and settling time of 1 sec.



Example - 2

2. Design a PI controller to the shown system which has desired poles of (S = -0.56±j0.4).